3) Consider an economy where

$$y_t = E_t y_{t+1} - s r_t + \epsilon_t^{IS}$$

$$\pi_t = E_t \pi_{t+1} + y_t + \epsilon_t^{AS}$$

$$r_t = a\pi_t + b y_t + \epsilon_t^{mp}$$

where *y* is the output gap (the log of output minus the log of natural-rate output), *r* is the interest rate gap (the real interest rate minus the natural rate of interest), and π is the inflation rate. The ϵ's are all uncorrelated with each other and mean-zero i.i.d. (*no persistence*). Expectations are rational.
Recall that we call a variable "procyclical" if it is positively correlated with the output gap; "countercyclical" if it is negatively correlated with the output gap, and "acyclical" if it is uncorrelated with the output gap.
For each case below, state whether each of the following variables is procyclical, countercyclical or acyclical: π, *r*, and the *nominal* interest rate *i*.

See the notes on NKIS/LM Simplest case: interest-rate rule, no persistence. (I said the shocks were i.i.d. and expectations are rational).Note that $E_t \pi_{t+1} = 0$ *always.*

The nominal interest rate is a bit tricky. Our usual definition for the real interest rate is $r_t = i_t - E_t \pi_{t+1}$. *On that definition the nominal interest rate is equal to the real interest rate. The answers I give below come from that assumption. But I didn't define the real interest rate here. It would not be unreasonable to define it as* $r_t = i_t - \pi_t$. *If you made that assumption, your answer to b) would be different with respect to the nominal interest rate (the cyclicality of the nominal interest rate would be ambiguous).*

3 pts each.

a) The variance of ϵ^{IS} is big, the variances of ϵ^{AS} and ϵ^{mp} very small. *A positive IS shock raises output, inflation and r. So all the variables are procyclical. (Nominal interest rate procyclical.)*

a) Starting from the Bellman equation, derive an equation that gives the agent's demand for real money balance $(M/P)_t$ as a function of consumption C_t and the nominal interest rate i_t. *3 pts.*

$$V_t = \max\left[\frac{1}{1-\theta} C_t^{1-\theta} + \frac{1}{1-\nu}(M/P)_t^{1-\nu} - \frac{1}{1+\lambda} L_t^{1+\lambda} + \beta E_t V_{t+1}\right]$$

$$\frac{\partial V}{\partial (M/P)_t} = (M/P)^{-\nu} + \beta E_t \frac{\partial V}{\partial A_{t+1}}(P - P(1+i_t)) = \ldots\ldots \quad P(-i_t) = 0$$

$$\frac{\partial V}{\partial C_t} = C_t^{-\theta} + \beta E_t \frac{\partial V}{\partial A_{t+1}}(-(1+i_t)P_t) = 0$$

$$\text{so } \beta E_t \frac{\partial V}{\partial A_{t+1}} P_t = C_t^{-\theta}\frac{1}{1+i_t} = (M/P)^{-\nu}\frac{1}{i}$$

$$\Rightarrow (M/P)_t = C_t^{\theta/\nu}\left(\frac{i}{1+i}\right)_t^{-1/\nu}$$

b) Starting from the Bellman equation, derive C_t as a function of $E_t C_{t+1}$ and the "real interest rate" $r_t = i_t - E_t \pi_{t+1}$, using $P_{t+1} = (1+\pi_{t+1})P_t$ and an approximation. *3 pts.*

$$V_t = \ldots\ldots$$

$$\frac{\partial V}{\partial C_t} = \ldots\ldots = 0$$

$$\frac{\partial V}{\partial A_{t+1}} = \frac{\partial u}{\partial C_{t+1}} \cdot \frac{\partial C_{t+1}}{\partial A_{t+1}} \quad \frac{1}{P_{t+1}} \text{ (holding fixed } A_{t+2})$$

(use certainty-equivalence

c) Suppose that long-run steady state consumption $\bar{C}$ grows at rate g (that is $\bar{C}_{t+1} = (1+g)\bar{C}_t$). Starting from your answer to b), derive the long-run steady state real interest rate $\bar{r}$. *3 pts.*

$\bar{C}_t = \bar{C}_{t+1}\left[\beta(1+\bar{r})\right]^{-1/\theta}$

$$Y_t = -\frac{1}{\theta}\ln\beta + E_t Y_{t+1} - \frac{1}{\theta} r_t + (1-\rho)\gamma_t$$

e) Suppose there is a central bank in this economy. Prior to each period t, prior to the realization of ϵ for that period, the central bank can set a value for the period's money supply M_t or the period's real interest rate r_t . Which would be better, assuming the central bank has a conventional loss function (the type of loss function we usually assume for a central bank)? Explain, using a graph or graphs as appropriate. *4 pts. Recall the Poole paper. In this economy there are no money-demand shocks, but there are IS(spending) shocks. So it would be better to fix the money supply. See graphs in*

notes.

5) In the Diamond-Dybvig model of liquidity/financial crises, there are two "types" of consumers. How are the two types different? *6 pts.*

There are "impatient" and "patient" consumers. They have different utility functions. Impatient consumers get utility from consumptin in the first period, but no utility from consumption in the second period. Patient consumers get utility from

consumption in both periods.

It is incorrect to say that impatient consumers withdraw in the first period, patient consumers leave their deposits in the bamk until the second period. This is incorrect because, in the event of a liquidity crisis, patient consumers also try to withdraw in the first period.

It is also incorrect to say that impatient consumers "prefer" to consume in the first period. That phrase does not imply that

4) Recall Romer's static imperfect-competition model. The utility function for the representative household consuming a continuum of goods *j* is:

$$U = C - \tfrac{1}{\gamma}L^{\gamma} \text{ where } C = \left[\int_{j=0}^{1} C_j^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}$$

Recall each good is produced by a monopoly firm that hires labor from a competitive labor market at a market-clearing nominal wage *W*, producing one unit of output from each unit of labor input. Recall that η ends up being the elasticity of demand for an individual good: $C_j = (P_j / P)^{-\eta} C$.

a) As a function of the model's parameters, what is the socially optimal value of *L*, that is the level of employment per household that would be chosen by a social planner who acts to maximize the representative household's utility *U?*

[redacted]

b) As a function of the model's parameters, what is the actual value of *L* in the model's "flex-price equilibrium," in which each household/firm's price is set at the profit-maximizing level, absent any type of nominal rigidity?

3 pts.See notes. Maximizing profit, each firm sets $P_j = \mu W$ *where* $\mu = \frac{\eta}{\eta-1} = \frac{1}{1-\eta}$ *. With all firms identical so that*

$P = P_j$,

that means $W / P = 1/\mu$.

[redacted]