Prof. uhn

Question 1 (60 points)

A farmer can roduce out ut in one of two ways, either with or without a cow. If the cow is alive, the roudction function is given by $y = Al$ where l is labor in ut. If the cow is dead the farmer roduces with a roduction function of $y = \underline{A}l$ with $A > \underline{A}$.

The out ut roduced in any eriod can be s lit u between consum tion c and feed f for the cow. The farmer derives utility from consum tion, and disutility from labor; s ecifically, the farmer's references are described by $u \quad c \quad v\frac{l^2}{2}$ where $u(\ )$ is a strictly increasing and strictly concave function and v is a scalar arameter.

The benefit of using the out ut for feed is that it increases the likelihood that the cow stays alive: The robability that the cow survives into the next eriod is given by (f) where $(\ )$ is an increasing and concave function with $(0) = 0$.

Denote with V^a the farmer's value function when the cow is alive, and with V^d the value function when the cow is dead.

1. Write down the dynamic rogramming roblem for this farmer. What are the state variables, what are the control variables? (15)

2. Write down the first-order conditions for the choice of l for both a live and dead cow. (15)

of $y = zn$ where n is the amount of labor su lied by the household and z is a roductivity shock. The household's labor su ly equals the total time endowment, which is 1, minus the chosen leisure time.

The roductivity shock takes on one of two ossible values in each eriod so that we can write $z \; 2 \; fz_H; z_L g$. The res ective robabilities are ^{H} and $^L = 1 \quad ^H$. The rocess is iid over time (which im lies that z_2 and z_3 are inde endent random variables), and roductivity in the first eriod is $z_1 = z_H$ with certainty.

Su ose that the economy has an Arrow-Debreu market structure, in which households and firms trade at time zero.

1. Carefully define a com etitive equilibrium for this economy. How many markets are there, and how many rices? Is the equilibrium allocation in this economy Pareto o timal? (20)

2. What is the risk-free interest rate at time 2 (that is, the interest rate between dates 2 and 3), conditional on the realization of z_2? What is the rice of a bond bought at time 2 in the state z_H that ays one unit of consum tion at time 3 in state z_L?
   (Hint: Proceed in two ste s. First, take the Arrow-Debreu rices as given. Ex ress the risk-free rate and the bond rice as a function of these rices. Second, use the household's o timality conditions to substitute out the rices, thereby relating interest rate and bond rice to arameters and equilibrium quantities.) (20)

3. Sim lify the setu in two ways. First, assume that there are only two e- riods in the model (that is, ignore the third eriod). Second, su ose that $u(c; l) = \log c + \log l$. Solve for all the equilibrium rices and quantities in this economy. (20)