Prof. uhn, Question 1

Consider a farmer whose cow has recently assed away. After a number of eriods on the farm, the farmer decides to switch jobs. He therefore moves to the nearest city and becomes a worker. U on arrival in the city the worker starts out unem loyed. In any eriod in which he is unem loyed, the worker receives unem loyment benefits of b.

In the city there are many otential em loyers. As long as the worker is unem loyed, he a lies to a new job and receives one new job offer every eriod. The only way in which job offers differ from each other is in the wage w aid by the em loyer; the distribution of wages is given by the differentiable cdf $F(w)$ with su ort w 2 $[w_{min}; w_{max}]$. The worker then decides whether to acce t or reject the job offer.

- If the worker rejects the offer, then next eriod he will still be unem loyed and a ly to a different otential em loyer, getting a new wage offer from the distribution F.
- If the worker acce ts the offer, then next eriod he will start working for the em loyer and from then on receive the wage w. The worker will then work for the same em loyer ermanently (for sim licity: em loyment lasts an infinite number of eriods).

The worker has a discount factor of $0 < < 1$ and does not save or borrow. The utility function $u(c)$ indicates how much utility the household derives from a given level of consum tion c. Denote the worker's value function when he is unem loyed with U and the value function when he has a job with V.

1. What are the state variable(s) for each of the two value functions? What is the choice the worker makes in each case?
2. Write down the value functions when the worker is em loyed and unem loyed, res ectively.
3. Find the utility value of em loyment by writing the value function for em loyment V in terms of its state variables and the arameters of the roblem. (In other words, find a non-recursive way to ex ress it.)
4. Using the result from 3. and the unem loyment value U, show that there is a cutoff value w for the wage offer such that the worker will acce t the job whenever he receives an offer greater than the reservation wage w w.
5. Characterize w as much as you can. Hint: You will have to write out the ex ected value of unem loy-ment.

Question 2

Consider the standard neoclassical growth model without labor. That is, there is a unit measure of identical households who derive utility from consum tion according to the lifetime utility function

$$E \left[\sum_{t=0} {}^{t} u(c_t) \right]$$

where the discount factor $2 (0; 1)$. Households own ca ital k_t and su ly 1 unit of labor inelastically. In every eriod t, they rent out these factors of roduction to firms and receive real factor ayments r_t and w_t for every unit of ca ital and labor rovided, res ectively. Ca ital de reciates at rate $2 (0; 1]$. In addition, households can trade a one- eriod risk-free bond b_{t+1} among each other (there are no initial bond holdings, ie $b_0 = 0$). Such a bond can be bought (or sold) at rice p_t at time t, and ays off one unit of the good at time $t + 1$ for sure.

A re resentative, rofit-maximizing firm uses the in ut factors ca ital K_t and labor L_t to roduce a single out ut good Y_t according to $Y_t = A_t F(K_t; L_t)$ where F is a constant returns to scale roduction function and A_t is aggregate roductivity following a stochastic rocess over time.

1. Define a com etitive equilibrium in this economy (be sure to write down all com onents ex licitly).
2. Characterize the equilibrium by deriving the first-order o timality conditions for firms and households.
3. Find the Euler equation for the economy's ca ital stock. To do this combine the relevant equilibrium conditions such that ca ital is the only endogenous variable in the equation. (Hint: you may find it useful to use the fact that the firm is erfectly com etitive and o erates with constant returns to scale).

Now use the following arameter values: Ca ital de reciates fully with $= 1$, households have log utility $u(c) = \log c$, and the roduction function is $F(K; L) = K \ L^{1}$. The random variable A_t follows an i.i.d. two- oint rocess: In every eriod, with robability ^{h} roductivity is $A_t = A^h$, and with robability $1 \quad {}^{h}$ it is $A_t = A^l$, with $A^h > A^l$.

4. Show that the olicy function for ca ital is given as $g(k; A) = \quad Ak$.
5. What is the rice of the risk-free bond? Find a closed-form ex ression. Use it to find out how the bond rice de ends on: current roductivity A_t? The discount factor ? The current ca ital stock k_t? The robability of a good shock ^{h}?