Question 1 **A credit problem** Consider the rm problem entrepreneurs considering borrowing to nance their operations, and the problem of households who consider lending to the rms. The timing is as follows:

At $t = 0$, rms start with 0 capital. They can issue bonds b at a bond price of q in order to borrow. They use these funds to purchase machinery k. There are no other options of nancing investment; and

Question 1 A credit problem Solutions:

1. The firm's problem is to

$$\max_{b;k} E\left[z k^{\alpha} + (1-\delta) k - b\right] \text{ s.t. } k = qb$$

or

$$\max_{k} E\left[z k^{\alpha} + (1-\delta) k - k/q\right]:$$

The first-order condition is

$$\frac{1}{q} - (1-\delta) = E\left[\alpha z k^{\alpha-1}\right]$$
$$\frac{1}{q} - (1-\delta) = \alpha k^{\alpha-1} E[z]$$

2. Since in this setup the households are guarantueed payback of the bonds, the expectation over period-1 utility is trivial. The household's problem is to

$$\max_{c_0;c_1;b} u(c_0) + \beta u(c_1)$$
$$\text{s.t. } c_0 = e_0 - qb$$
$$c_1 = b$$
$$c_0 \geq 0$$
$$c_1 \geq 0$$

or

$$\max_{b} u(e_0 - qb) + \beta u(b) + \lambda_0 (e_0 - qb) + \lambda_1 b$$
$$\text{s.t. } \lambda_0 c_0 = 0$$
$$\lambda_1 c_1 = 0$$

The first-order condition for an interior solution (i.e. $\lambda_0 = \lambda_1 = 0$) is

$$u'(c_0)\, q = \beta u'(c_1)$$
$$q = \beta \frac{u'(c_1)}{u'(c_0)}:$$

3. With linear utility, the first-order condition becomes $q = \beta$: From firms' first-order condition we have

$$k = \left(\frac{\alpha E[z]}{\frac{1}{q} - (1-\delta)}\right)^{\frac{1}{1-\alpha}} = \left(\frac{q \alpha E[z]}{1 - q(1-\delta)}\right)^{\frac{1}{1-\alpha}}$$
$$= \left(\frac{\alpha \beta E[z]}{1 - \beta(1-\delta)}\right)^{\frac{1}{1-\alpha}}$$

with the equilibrium bond price substituted in.

4. From the Hhs FOC we know that the bond price is constant. We can see from the optimal investment condition in 3. that a mean-preserving spread does not affect optimal investment (at least if firm and households are both risk-neutral as we assumed). While the firm's expected profits are not affected, the firm's realized profits now depend on the realization of z. If $z = 0$ the firm is running a loss with realized profits $\pi = k[1 - \delta - 1/\beta]$. Because firms always repay the bond, household welfare is not affected.

5. First note that now repayments are $\min\{zk \;\; + (1 \;\;\;)k; b\}$. Since households now know that there is a risk of default, the problem becomes

$$\max_b u(e_0 \;\;\; qb) + \;\; E \;\; [u(\min\{zk \;\; + (1 \;\;\;)k; b\})]$$

with FOC

$$u^0(c_0)\,q$$

Question 2 An optimal timing problem A young couple has recently graduated and entered the job market. They currently rent an apartment, but their ultimate goal is to build a house. Both have steady jobs earning them a combined wage income of constant real wage $w_t = w$ every period, regardless of their living

Solutions:

1. The only state variable is the current living situation, i.e. a binary $\{rent; own\}$. The only choice is next period's living arrangement and because the household never goes from owning back to renting this really only applies to the renting situation. (We could include consumption as a choice variable which would be trivial since the budget constraint is just $c = w \quad r$.) With the description of the timing, the household problem can be expressed as

$$V^r = u(w \quad r) + \max\{ \; V^r; \quad + \; V^o\}$$
$$V^o = u(w) + \quad + \; V^o:$$

2. The question is, under which condition is the household going to choose renting over building & owning? That is, we are looking for the combinations of paramters such that

$$V^r > \quad + \; V^o:$$

The rst thing to note that if this condition is true in this period, then it is true for all periods (because renting/owning is the only state variable, so both sides of the inequality are constant). This means that we can already say something about the dynamics of the decision rule: Either the couple build a house right away, or never at all. We can therefore also write $V^r = u(w \quad r) + \; V^r \;) \quad V^r = u(w \quad r) = (1 \quad)$. For the value of owning we get $V^o = \frac{u(w)+}{1}$. Substituting in above, we have

$$\frac{u(w \quad r)}{}$$

because uctuates over time. The expectation is

$$
\begin{aligned}
E[V^r(\ )] &= E[u(w\ \ r) + \max f\ E\ [V^r(\ ^0)];\ \ +\ V^o g] \\
&= u(w\ \ r) + E[\max f\ E\ [V^r(\ ^0)];\ \ +\ V^o g] \\
&= u(w\ \ r) + \int_0 \ \ d\ \ +\ V^o + \int^1 E\ [V^r(\ ^0)]\, d \\
&= u(w\ \ r) + \int_0 \ \ d\ \ +\ V^o + \ 1\ \ F\ \ \ E\ [V^r(\ )] \\
E[V^r(\ )]\ \ 1\ \ \ 1\ \ F\ \ &= u(w\ \ r) + \int_0 \ \ d\ \ +\ V^o \\
E[V^r(\ )] &= \frac{u(w\ \ r) + \int_0 \ \ d\ \ +\ V^o}{1\ \ \ 1\ \ F}:
\end{aligned}
$$

Substituting this into the above gives

$$
\begin{aligned}
\frac{u(w\ \ r) + \int_0 \ \ d\ \ +\ V^o}{1\ \ \ 1\ \ F} &> \ \ -+ V^o \\
u\ (w\ \ r) + \int_0 \ \ d\ \ +\ ^2V^o &> \ 1\ \ \ 1\ \ F\ \ \ \ -+ V^o \\
u\ (w\ \ r) + \int_0 \ \ d\ \ +\ 1\ \ \ 1\ \ F\ \ \ - &> V^o\ \ 1\ \ \ 1\ \ F\ \ \ +\ \ ^2 \\
u\ (w\ \ r) + \int_0 \ \ d\ \ +\ 1\ \ \ 1\ \ F\ \ \ - &> \frac{u(w) +}{1}\ \ 1\ \ \ 1\ \ F\ \ \ +\ \ ^2:
\end{aligned}
$$

Note that these are all constants except for which is the random variable. The condition implicitly de nes a such that if the inequality no longer holds (the r.h.s. becomes less or equal than the l.h.s.).

5. We can see that if the household is very impatient the utility cost becomes very important as= ! 1 , and the condition will hold for sure (and household rents). Conversely, if the household becomes very patient then the value of owning a home $V^o = \frac{u(w)+}{1}$ dominates, makes the r.h.s. arbitrarily large so that the condition fails and Hhs build for sure.