

1 Part A: Short answer or explanation. 4 questions.

1. (Jones) Calculate the elasticity of substitution for the production function

$$f(x_1; x_2) = k(1 + x_1^{0.5}x_2^{0.5})^{-1}$$

where $k > 0$.

2. (Jones) A consumer has lexicographic preferences over \mathbb{R}_+^2 . I.e., $x^1 \succ x^2$ if and only if $x_1^1 > x_1^2$, or $x_1^1 = x_1^2$ and $x_2^1 > x_2^2$. Sketch and indifference map for these preferences. Do these preferences satisfy the continuity axiom? Explain.

3. (Pape) Find all Nash Equilibria of the following game:

	L	C	R
T	0; 5	6; 7	6; 3
M	0; 8	2; 6	9; 4
B	2; 2	4; 6	1; 1

4. (Pape) What is the first fundamental theorem of welfare economics and why is it true? You need not provide a complete proof; a concise argument is sufficient.

2 Part B: Mathematical or advanced questions. 6 questions.

5. (Jones) A consumer of 2 goods has an indirect utility function given by
- $v(p_1; p_2; y) = \frac{y}{p_1 + p_2}$
- , where
- p_1
- and
- p_2

8. (Pape) Frank and Johnny play a game of Chicken on bicycles every Sunday on the Exchange Street bridge in downtown Binghamton. Frank and Johnny have two identical actions which yield the same payoffs. The actions are Tough and Chicken. The payoffs are: $u(C;C) = 1$, $u(C;T) = 0$, $u(T;C) = 3$, and $u(T;T) = -1$.
- (a) Write down the normal form of this game. Find all NE of this game.
- (b) The weather in Binghamton is Rainy with probability θ and it is Sunny with probability $(1 - \theta)$. Find all θ for which there is a Bayes-Nash Equilibrium (or "Perfect Bayesian Equilibrium") in which (1) the payoffs are symmetric across players in expectation and

3 Part C: Longer questions. 4 questions.

11. (Jones) A consumer lives 2 periods, $t = 0; 1$. Let x_t denote consumption spending in period t , y_t denote income in period t , and $r > 0$ the market interest rate. The consumer's intertemporal utility function is given by

$$u^2(x_0; x_1) = \frac{1}{2}(x_0 - 2)^2 + \frac{1}{2}(x_1 - 2)^2$$

where $0 < \beta < 1$. The intertemporal budget constraint requires that the present value of expenditures not exceed the present value of income.

- (a) If $y_0 = 1$, $y_1 = 1$, and $\beta = \frac{1}{1+r}$, solve for optimal consumption in each period and calculate the consumer's lifetime utility.
- (b) Suppose, now, that the consumer knows that income in the initial period will be $y_0 = 1$.

13. (Pape) Consider the following game. There are n individuals, numbered $i = 1, \dots, n$, who each have one dollar. (Assume utility is linear in money.) When the game begins, each person simultaneously chooses a fraction $x_i \in [0, 1]$ of their wealth to throw into a *magic well*. After each agent has selected an x_i , all x_i 's are revealed, and the magic well produces a payoff for each player. The award is the same for each player, regardless of contribution. The award is:

$$a = d \frac{\sum_{i=1}^n x_i}{n}$$

where $d > 0$.

- Find all Nash Equilibria of this game, as a function of d .
- This is a game theory problem. However, the magic well in this problem is an example of a concept we encountered when we studied Walrasian equilibrium. What is that concept? Why do you think so? Explain!

For part (c), consider these two modifications to the game:

- Let $d = 5$.
- Assume that there is a second round of play which occurs just after the magic well gives its awards, in which each agent i simultaneously chooses a number $x_i \in [0, 1]$. Then, the x_i 's are revealed, and any agent whose number is revealed (i.e. any i such that $x_i = 0$) immediately loses a dollar. (Note that no one has $x_i = 0$, which