Graduate Program in Economics, Binghamton University

ECON 603: Advanced Mathematical Analysis for Economists

Diagnostic Exam

Question 1

a) With reference to the function $f : R \quad R$,

i. State the Mean Value Theorem of the differential calculus (MVT);
ii. State the Taylor Series Expansion of $f(x)$ about the point $x \quad a$

Carefully demonstrate that the MVT is a special case of the Taylor Series.

b) Use the Taylor Series to prove that $f'(a) \quad 0$ is a necessary condition for the function f to have a relative extrema at (a, f(a)). Furthermore, prove that if $f'(a) \quad f''(a) \quad 0$ and $f'''(a) \quad 0$, then the point (a, f(a)) is neither a relative minimum nor a relative maximum point.

c) By considering the Taylor Series expansion of $f(x) \quad e^x$ about $x \quad 0$, show that

$$e \quad 1 \quad 1 \quad \frac{1}{2} \quad \frac{1}{6} \quad R$$

Where $R \quad \frac{1}{8}$

Question 2

Suppose that aggregate consumption in period t, C_t is a linear function of aggregate income in the previous period, Y_t, that is,

$$C_t \quad A \quad BY_{t \quad 1}$$

Question 3

Consider a representative agent consumes two goods, economic books and bread. Her utility function is given by $U(X,Y) \quad X \quad Y$, where X stands for the number of economic books and Y stand for the number of bread. and are such that 1.

a) By substituting for e_t in equation (*) and then eliminating $e_{t\ 1}$ from the resulting equation, show that

$$y_{t\ 2} \quad (\qquad) y_{t\ 1} \qquad y_t \quad u_{t\ 2}$$

b) Given that $\qquad \frac{1}{2}$ and $u_t \quad 0$, solve the difference equation that you obtained in part (a).

Question 6

Given the following non-linear maximization problem:

$$\text{Maximize} \quad Z \quad 2x_1^2 \quad 8x_2^2$$

$$\text{Subject to} \quad \begin{array}{l} x_1^2 \quad x_2^2 \quad 16 \\ x_1, x_2 \quad 0 \end{array}$$

a) Formulate the Lagrangian function for this problem.